Reviewing Comparison Tests

This was a good question that I think many of you might have in the backs of your minds. Read this over and see if it helps. If it doesn't help, or if it raises more questions, please ask right away. This is a tough subject and talking over confusing bits is the only way to master it.

I think I understand the problem I had with the last section, but in this section, I'm getting confused about which test to indicate in the conclusion.
That really is normal - we have so many tests that it takes a while to get it all straight.

When I started on the homework it was making sense at first but when I got to the part about absolute and conditional convergence, it was confusing me and making me second guess the work I already did. I'm not sure if this is right, but if you have an alternating series and you use the nth term test to show it diverges, then you don't have to say it diverges by the alternating series test but instead mention the nth term test? **Yes, that is right.**
The Alternating Series Test (AST) only can prove that your alternating series converges. One of the parts of the Alternate Series Test is to show that the terms are approaching zero - which is really the n^{th} term test. For the AST you are showing that the series "passes" the n^{th} term test. If it fails it, it is actually failing the n^{th} term test, not the AST.

But if you're proving a convergent alternating series, you state it is because of the alternating series test?
Yes, because the AST says that if A (nth term test shows that terms are approaching 0) and B (terms are decreasing) are both true, then the series must converge (and we say the reason is because of the AST).

I guess overall what I'm really not sure about is when we do these problems, we're really doing the absolute of the series (is that right?) because we're ignoring the (-1)^n part of the series.
Well, yes and no. I guess we are ignoring the $(-1)^n$ but that does not mean we are showing that the absolute value series is converging.

For an alternating series, the only two conditions that need to be met for it to converge are for the terms to approach 0 and to decrease.

That is not enough for an absolute value series, a series whose terms do not alternate in sign. A case in point is the harmonic series, the p-series where $p = 1$. The terms of $1/n$ approach zero and they are decreasing. However, it diverges.

For a series that has terms of all one sign (all positive or all negative), we have to use one of our other tests (Direct Comparison, Limit Comparison, Ratio, Root, Integral, geometric, p-series).

But then the book talks about conditional convergence and how a divergent absolute is conditional and stuff. I don't know, the more I look at it and think about it the more I get confused.
What they are saying is that when we ignore the $(-1)^n$ and prove the absolute value series is convergent using one of our other tests, then the alternating series will also converge and that we will call this Absolute convergence.

If we cannot do that (we try but find out that the absolute value series diverges), then we can go back to the Alternating Series Test and try to show that the alternating series converges. When this holds true, then the alternating series converges but the absolute value series does not - this is conditional convergence (a term we only apply to alternating series).

When an alternating series does not converge it is usually because it fails the n^{th} term test (its terms do not approach zero).

In summary:

For <u>absolute convergence</u> we show the absolute value series converges (using one of the other tests), and therefore it follows that the absolute value series also converges. (it is "easier" for an alternating series to converge; its terms kind of cancel each other out because of the opposing signs so it is easier for the sum to not grow without bound). The absolute value series is the series we have when we take the alternating series and make all the terms positive instead of alternating in sign.

For <u>conditional convergence</u>, we try, but fail, to show that the absolute value series converges. Thus we have to "retreat" to the Alternating Series Test and show that our alternating series converges. When doing this we sort of ignore the (-1)^n, but that does not mean that we are testing the absolute value series. We are just looking at the behavior of the a_N that will be alternating in sign.

For <u>divergence</u>, both attempts of showing convergence failed. If the part that fails is the n^{th} term test, then both the alternating and the absolute value series diverge.

Note – you do not have to test an alternating series for absolute convergence unless you are asked to! If the question does not ask for that extra step, then you can go straight to the AST and just use it to prove (conditional) convergence.